Linear Functions and Average Rate of Change

Section 2

Math 111, W22

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Motivating Example: Trails at Crater Lake NP

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- The Mount Scott Trail begins at an elevation of 7500 feet and ends 2.5 miles later at an elevation of 9000 feet.
- You are feeling up for a challenge and wish to do the steepest trail. Which trail should you hike?

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- Intuitively, you say "how much am I changing altitude over the distance that I'm hiking?"
- A little more formally, you might say
 - Cleetwood Cove has a total elevation change of 7900 7200 = 700 feet and hence, an average elevation change of $\frac{700}{1.2} = 583.\overline{33}$ feet per mile.

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So you should take the Mount Scott trail!

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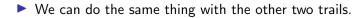
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 - Elevation 1.2 miles from the trailhead is c(1.2), so c(1.2) = 7900 feet.
 - Our earlier computation to figure out "steepness" was exactly $\frac{c(1.2)-c(0)}{1.2-0} = 583.33$ feet per mile.



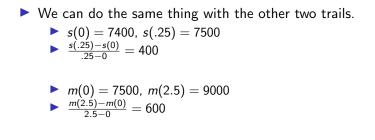


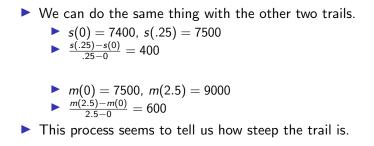
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$$m(0) = 7500, \ m(2.5) = 9000$$





Formal Definition

This motivates the following definition.

Definition: The Average Rate of Change of a function Q = f(t) on the interval [a, b] is

$$ARC_{[a,b]} = \frac{f(b) - f(a)}{b - a}$$

This rate of change is frequently expressed as

$\frac{\Delta Q}{\Delta t}$

where ΔQ is the total change in the output Q and Δt is the total change in the input t over the interval [a, b].

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$$ARC_{[3,8]} = \frac{f(8) - f(3)}{8 - 3} = \frac{\sqrt{9} - \sqrt{4}}{5} = \frac{3 - 2}{5} = 0.2.$$

Back to our analogy of a trail, we know exactly what it means for our elevation to be strictly increasing, decreasing, or constant.

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 - A trail is strictly increasing in elevation if, at all points on the trail, you are going up. (i.e. there aren't any stretches of trail where you aren't going up)
- ▶ We can rephrase these conditions as:
 - A trail is strictly increasing in elevation if every stretch of trail is going up.

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Increasing, Decreasing, and Constant Functions

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 - A trail is strictly increasing if for every interval [a, b] in the practical domain, the elevation at b is greater than the elevation at a.
- But we have a way of phrasing "the elevation at b" a little bit more mathematically
 - A trail is strictly increasing if for every interval [a, b] in the practical domain, f(b) > f(a).

Increasing Decreasing and Constant Functions

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Definition: A function f(t) is Strictly increasing on the constant
 interval (c, d) as long as f(b) > f(a) f(b) < f(a) f(b) < f(a) f(b) < f(a) f(b) = f(a)
 values a < b in the interval [c, d].

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- Definition: A linear function is any function with the property that for any pair of points on the graph of the function, the average rate of change between those points has the same value, a. This value is called the *slope* of the function. Such a function can always be written

$$f(t) = at + b$$

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for some numbers *a* and *b*. The number *b* is called the *intercept*.

Sanity check: How do we know that any function of the form f(t) = at + b has a constant average rate of change?

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 Pick any interval you like, let's say [c, d].
 The average rate of change on that interval is

$$ARC_{[c,d]} = \frac{f(d) - f(c)}{d - c} = \frac{(ad + b) - (ac + b)}{d - c}$$
$$= \frac{ad - ac}{d - c} = \frac{a(d - c)}{d - c} = a.$$

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$$= \frac{ad - ac}{d - c} = \frac{a(d - c)}{d - c} = a.$$

Since this average rate of change does not depend on the choice of c, d it is always constant, agreeing with what we said earlier.

▶ What do slope and intercept actually *mean*?

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What do slope and intercept actually mean?

- Let's say you know that a linear function has slope a. This tells you that if the input to the function increases by 1, then the output increases by a.
 - Example: if I'm looking at a linear function, a(I) with slope 2, I know that a(1) will be 2 greater than a(0)

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 - Example: if I'm looking at a linear function, a(1) with slope 2, I know that a(1) will be 2 greater than a(0)
- Let's say you know that a linear function has intercept b. This tells you that when the input is 0, the output will be b.
 - **Example**: if I'm looking at a linear function a(1) with intercept -3, I know that a(0) = -3

- How do we actually solve problems about linear functions?
- Most problems that you can be asked will give you some kind of information, then ask you to find the equation of a linear function that satisfies that information.

- So how do you find the equation of a linear function?
- Note that finding a linear function requires two pieces of information
 - 1. Slope
 - 2. Intercept

How do we find slope?



How do we find slope?

If the average rate of change is given to you, the slope is exactly the average rate of change.

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How do we find slope?

- If the average rate of change is given to you, the slope is exactly the average rate of change.
 - Example: The number of rabbits in a population was increasing at a rate of 25 rabbits per year. What is the slope of the function which represents the number of rabbits as a function of year?

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Answer: -9 foxes per year

• (Recall: For a point on a graph (t_1, y_1) , $y_1 = f(t_1)$.)

If you are given two points on the graph of the function f, say (t₁, y₁) and (t₂, y₂), then the slope is just the average rate of change between those points,

$$\frac{y_2-y_1}{t_2-t_1}$$

▶ Example: Say we know that (-1,5) and (2,3) are points on the graph of a linear function. The slope of that function is

$$\frac{3-5}{2-(-1)} = \frac{-2}{3}$$

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 - If you know the slope and you know a single point on the graph, you can solve for the intercept.
 - Example: Suppose that weight is a linear function of age with slope 1.1 pounds per month. If a ten month old child weighs 20 pounds, what is the intercept of that function?

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Let's do it together.





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- So, the question is, what do these things mean on a graph?

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- If you know anything at all about slope, it's that the more positive the slope gets, the more steeply upwards the graph points. The more negative the slope gets, the more steeply downward the graph points.

Collinearity

- Definition: We say that a set of points (t₁, y₁), (t₂, y₂), ..., (t_n, y_n) is *collinear* if there is a vertical line or single linear function whose graph contains all n points, i.e. if a single line passes through all of the points.
- How do we find out if a set of points is collinear? Find the line that goes through two of the points, then check to see if the other points lie on that line!
 - Finding all of the possible lines, then checking to see if they're the same is a lot of work.

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 If we start with a nonlinear Graph of n: function, n(x), we can compute the average rate of change between a, b by ^{n(b)-n(a)}/_{b-a}, which is the exact same formula as the formula for slope between the points (a, n(a)) and (b, n(b)).

Synthesis (continued)

- So the average rate of change between two points on the graph of a function is exactly the slope of the line between the two points.
- This is really important in calculus, where the entire goal is to approximate functions by straight lines.
- Every "nice" curve has the property that, if you zoom in far enough, it looks like a straight line. (Example: the Earth is actually curved, even though it looks flat from our perspective.)